ANALYSIS I BACKPAPER EXAMINATION

Total marks: 100
Time: 3 hours
Attempt all questions. If you use a result proved in class, please quote it completely and clearly.

(1) Let A be a countable set, and let $F(A)$ denote the collection of all finite subsets of A. Is $F(A)$ countable? Justify your answer. (10 marks)
(2) Calculate the limit $lim_{n\to\infty} \frac{2^n}{n!}$. (10 marks)
(3) Prove that (x_n), where $x_n = \sum_{i=1}^{n} 1/i!$, is a Cauchy sequence. (10 marks)
(4) Does the sequence (x_n), where $x_n = 2^{3n}/3^{2n}$ converge? Justify your answer. (10 marks)
(5) Let $f : \mathbb{R} \to \mathbb{R}$ be a function, let I be an interval contained in $\mathbb{R}$, let g denote the function f restricted to I, let $c \in I$. Consider the following statement: f has a limit at c, if and only if, g has a limit at c, and these two limits are the same (if they exist). Is this statement true if I is any open interval? Is this statement true if I is any closed interval? (15 marks)
(6) Give an example, with all details, of two uniformly continuous functions on $\mathbb{R}$ whose product is not uniformly continuous. (15 marks)
(7) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = 0$ if x is irrational and $f(m/n) = 1/n$ where $m \in \mathbb{Z}$, $n \in \mathbb{N}$ and $(m, n) = 1$. Prove that $lim_{x\to c} f(x) = 0$ for all $c \in \mathbb{R}$. (15 marks)
(8) Let $f : I \to \mathbb{R}$ be an increasing function on an interval I. Suppose $c \in I$ is not an endpoint of I. Prove that f is continuous at c if there exists a sequence (x_n) in I such that $x_n < c$ for n odd, $x_n > c$ for n even and such that $lim_{n\to\infty}(x_n) = c$ and $lim_{n\to\infty}(f(x_n)) = f(c)$. (15 marks)

Date: December 6, 2015.